2025 WAMC 10 Problems

1. What is the value of
$$\frac{2025 + |2025 - (2025 - 2025)| - 2025}{2025}?$$
(A) 0 **(B)** 1 **(C)** 2 **(D)** 5 **(E)** 2025

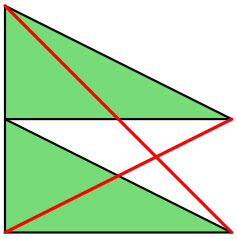
2. Points A, B, and C lie on circle ω with center O. If $\angle OAB = \angle OAC = 20°$, find $\angle OBC$.
(A) $10°$ **(B)** $20°$ **(C)** $40°$ **(D)** $50°$ **(E)** $70°$

3. In a game, Alex has a perfectly spherical apple. He first places down a basic sprinkler, which increases the radius of the sphere by 50 percent. Then, he places down an advanced sprinkler, which further increases the radius by 100 percent. What is the percent increase of the volume of the new apple compared to the original?
(A) 2600 **(B)** 2700 **(C)** 2800 **(D)** 2900 **(E)** 3000

4. Arjun, Vincent, Jacob, and Alexander stand in a line. Arjun refuses to stand in the first or second position. Vincent refuses to stand in the second or third position. Jacob refuses to stand in the third or fourth position. Alexander refuses to stand in the fourth or first position. How many possible orderings are there?
(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

5. What is the maximum amount of intersections between 2 circles and 1 triangle?
(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

6. In the figure below, the sum of the squares of the lengths of the red segments is 10. The two congruent light blue right triangles have leg lengths in the ratio $1 : 2$. Find their total area.



(A) $\frac{6}{7}$ **(B)** $\frac{15}{13}$ **(C)** $\frac{9}{7}$ **(D)** $\frac{10}{7}$ **(E)** $\frac{20}{13}$

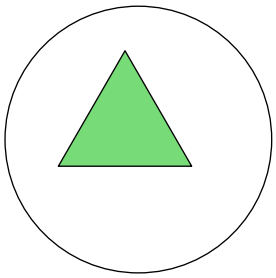
7. Ryan writes down the number 11111122. Patrick randomly rearranges the digits into a new eight-digit number. What is the probability that the sum of the digits in the left half equals the sum of the digits in the right half?
(A) $\frac{1}{4}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{5}{7}$

8. Squares $ABCD$, $DEFG$, and $GHIA$, each of side length 3, are drawn such that each square has no overlap with triangle ADG. Suppose each vertex of hexagon $BCEFHI$ lies on a circle. What is the area of the circle?
(A) $12\pi - 3\pi\sqrt{3}$ **(B)** 12π **(C)** $12\pi + 3\pi\sqrt{3}$ **(D)** $12\pi + 6\pi\sqrt{3}$ **(E)** $12\pi + 9\pi\sqrt{3}$

9. Let p be a prime for which there exist positive integers a and b satisfying $\sqrt{a^2+p}=b^2$. Find the sum of the four smallest possible values of p.

(A) 104 **(B)** 116 **(C)** 126 **(D)** 138 **(E)** 144

10. Laura takes a sponge in the shape of an equilateral triangle with side length 2 and dips it in paint. Then, without rotating the sponge or painting outside the circle, she paints as much of the interior of a circle with radius 2 as she can. How much of the circle is left unpainted?



(A) $2\pi-3\sqrt{3}$ **(B)** $2\pi-3\sqrt{2}$ **(C)** $2\pi-2\sqrt{3}$ **(D)** $4\pi-3\sqrt{3}$ **(E)** $4\pi-2\sqrt{3}$

11. Ryan places a standard fair 6-sided die (with opposite faces summing to 7) on the table with the 1 face on top. He then rolls the die to an adjacent face three times in succession. What is the probability that no number greater than 3 ever appears on the top?

(A) $\frac{1}{216}$ **(B)** $\frac{1}{64}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{1}{8}$ **(E)** $\frac{1}{6}$

12. Five cubes with side lengths 2, 3, 1, 5, 4 are stacked in that order, where the centers of touching sides align. What is the surface area of that tower (including the bottom face)?

(A) 270 **(B)** 275 **(C)** 278 **(D)** 286 **(E)** 330

13. What is the area formed by all coordinate pairs (x,y) that satisfy

$$(x-x_1)^2+(y-y_1)^2\le 4$$

over all x_1 and y_1 that satisfy $x_1^2+y_1^2=200$?

(A) $80\pi\sqrt{2}$ **(B)** $100\pi\sqrt{2}$ **(C)** 200π **(D)** 204π **(E)** $160\pi\sqrt{2}$

14. Four consecutive natural numbers a,b,c,d satisfy $a<b<c<d$. If a is divisible by 3, b is divisible by 5, c is divisible by 7, and d is divisible by 9, find the minimum value of $a+b+c+d$.

(A) 82 **(B)** 106 **(C)** 510 **(D)** 642 **(E)** 654

15. Real numbers x and y satisfy the system of equations

$$\begin{aligned} x^3-8x^2+px-q&=0,\\ y^3-8y^2+py-q&=0, \text{ and}\\ (x+y)^3-8(x+y)^2+p(x+y)-q&=0. \end{aligned}$$

There exists a unique k for which $kp+q$ is constant for all values of x and y. Find the value of $kp+q$.

(A) -64 **(B)** -58 **(C)** -32 **(D)** -10 **(E)** 26

16. Triangle $\triangle ABC$ intersects circle ω at points D, E, F, G, H, I such that $ADEB$, $BFGC$, and $CHIA$ are collinear in those orders. Given that $AD = 3$, $AI = 4$, $HC = 4$, $GC = 5$, and $DE + FG = 8$, find the value of DE.

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

17. There are 8 crabs and a box arranged in a line, with 5 crabs to the left of the box, and 3 crabs to the right of the box. Every second, a crab neighboring the box jumps into the box and disappears. What is the probability that the rightmost crab is the last one to disappear?

(A) $\frac{15}{128}$ **(B)** $\frac{21}{128}$ **(C)** $\frac{29}{128}$ **(D)** $\frac{3}{8}$ **(E)** $\frac{51}{128}$

18. In a math competition with 100 participants ranked from 1 to 100, Ryan randomly chooses an integer $N > 4$, then randomly interviews 2 of the top N participants. What is the probability that both the first and second-ranked participants are interviewed?

(A) $\frac{1}{2500}$ **(B)** $\frac{1}{1000}$ **(C)** $\frac{1}{500}$ **(D)** $\frac{1}{200}$ **(E)** $\frac{1}{100}$

19. How many ways are there to rearrange the letters in the word NAGGING so that no G's are adjacent?

(A) 48 **(B)** 60 **(C)** 120 **(D)** 168 **(E)** 240

20. How many ordered pairs of integers (m, n) satisfy the equation

$$\sqrt{m} + \sqrt{m + 100} = \sqrt{n}?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** Infinitely many

21. For how many integers n between 1 and 100, inclusive, is $\frac{n!^5}{5^n}$ an integer?

(A) 25 **(B)** 30 **(C)** 50 **(D)** 75 **(E)** 80

22. Bruce has 55 dice, each numbered 1 through 10. Each face of every die is weighted so that the probability of rolling n is directly proportional to n. Patrick rolls each of the die, and he allows Patrick to reroll each die once if he wants. Patrick wants to maximize the total sum of the dice and plays optimally. What is the expected value of this sum?

(A) 385 **(B)** 437 **(C)** 441 **(D)** 464 **(E)** 505

23. Isosceles trapezoid $ABCD$ has $AB \parallel CD$ and $AB = 3$. Triangle ABE is drawn outside the trapezoid with $\angle ABE = 90°$. If $EC = 25$, $ED = 26$, and the area of pentagon $AEBCD$ is 104, compute the area of triangle BDE.

(A) 54 **(B)** 76 **(C)** 80 **(D)** 104 **(E)** 204

24. How many ordered pairs (a, b) are there to the equation

$$\gcd(a, b)^3 + 90 \gcd(a, b) = (\gcd(a, b) + 1)(b - a)$$

if $1 \leq a \leq 500$?

(A) 42 **(B)** 43 **(C)** 70 **(D)** 98 **(E)** 129

25. A fair six-sided die is rolled repeatedly. Let m_k and M_k be the minimum and maximum rolls, respectively, after k rolls of the die. Let n be the least possible n such that $M_n - m_n \geq 4$. Find the expected value of n.

(A) $\frac{7}{2}$ **(B)** 4 **(C)** 5 **(D)** $\frac{23}{4}$ **(E)** 6